MAGNETIC FIELD RESPONSE IN FERRONEMATIC CELLS

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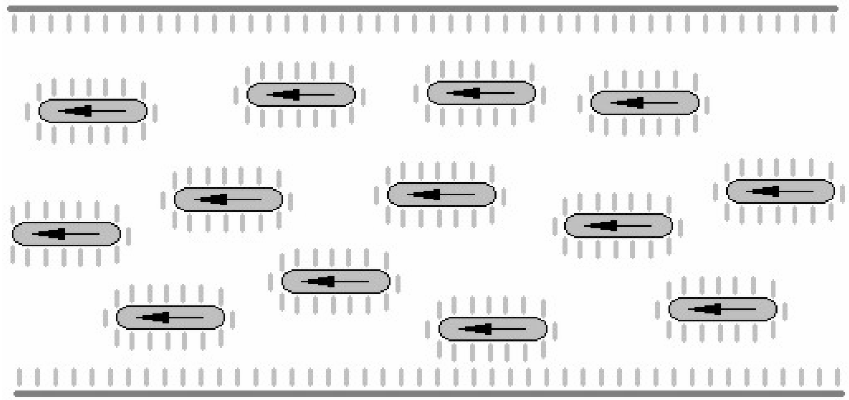
Ferronematics

- Magnetic colloids: “ferroparticles”
- Nematic liquid crystal matrix
- Magnetic and nematic orientation coupled
- Can exhibit a giant director response to a magnetic field even at very low colloidal concentrations.
- Magnetic Frederiks effect at accessible magnetic fields

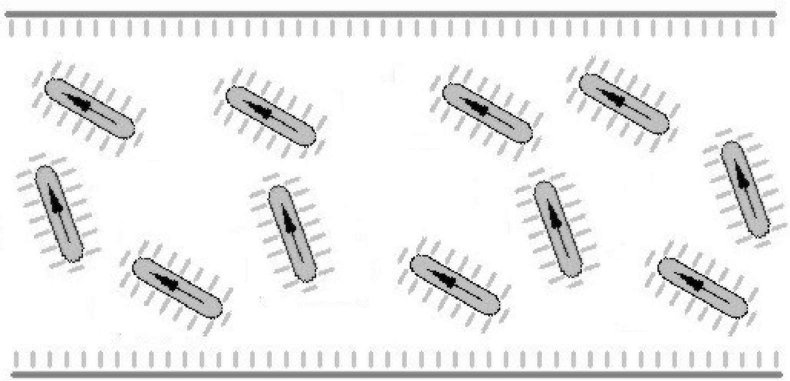
THE SYSTEMS



H=0

Note director orientation,

Arrows:
Ferroparticle orientation



$H \neq 0$

Field perpendicular to cell surface

Magnetic field indirectly reorients nematic director

SOME HISTORY

Theoretical prediction by **Brochard and de Gennes (1970)**

Experimental verification by **Chen and Amer (1983->)**
but suspensions were not stable

Lyotropic liquid crystal matrix **Figueiredo Neto *et al* (1984 ->)**

Theory **Burylov, Raikher and coworkers (1986->)**

Stable suspensions **Buluy et al (2002) (Kiev)**

Goal of this work

Examine Frederiks-like effects

1. Field
2. Bias Field
3. Colloidal volume fraction $f(z)$
4. Nematic-ferroparticle coupling W_p

Need to **systematise** previous work

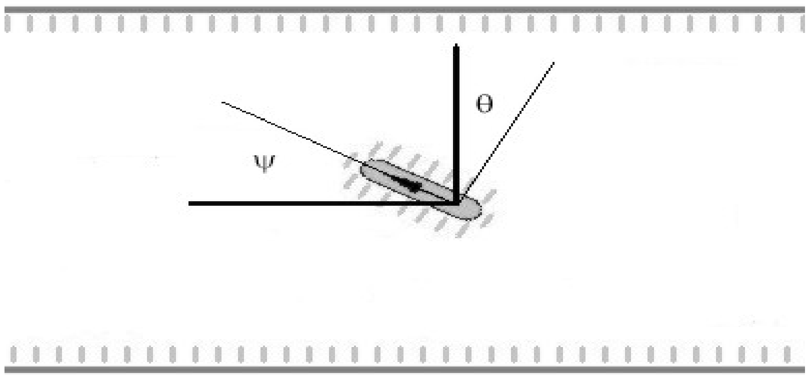
FREE ENERGY

$$F = \int_0^L \left\{ \frac{1}{2}\left[K_1 (div\,\mathbf{n})^2 + K_2 (\mathbf{n}.curl\,\mathbf{n})^2 + K_3 (\mathbf{n} \times curl\,\mathbf{n})^2 \right] - \frac{1}{2}\chi_a (\mathbf{n}.\mathbf{H}_s)^2 - M_s (\mathbf{m}.\mathbf{H}_s) f(z) + f(z) W_p (\mathbf{n}.\mathbf{m})^2 + \frac{f(z) k_B T}{\nu} \ln f(z) \right\} dz$$

$$F=\int_0^D\left\{\frac{1}{2}\left[K_1(div\,\mathbf{n})^2+K_2(\mathbf{n}.curl\,\mathbf{n})^2+K_3(\mathbf{n}\times curl\,\mathbf{n})^2\right]-\frac{1}{2}\chi_a(\mathbf{n}.\mathbf{H}_s)^2-M_s(\mathbf{m}.\mathbf{H}_s)f(z)\right.$$
$$\left.+f(z)W_p(\mathbf{n}.\mathbf{m})^2+\frac{f(z)k_BT}{\nu}\ln f(z)\right\}dz$$

1. Nematic elastic energy
2. Direct magnetic field-nematic interaction (negligible)
3. Magnetic field-ferroparticle interaction
4. Anchoring-mediated ferroparticle-director coupling W_p
5. Entropy associated with ferroparticle density *f(z)*
   ν is ferroparticle volume
6. $\boldsymbol{H}_s$=Imposed field $\boldsymbol{H}$(||)+ Bias field $\boldsymbol{H}_b$(=)

TECHNICAL STUFF



Director distortion profile (angle θ)

Ferroparticle orientation (angle ψ)

$$f(z) = \bar{f}\eta(z)$$

$\bar{f}$ is mean volume fraction

$$\eta = 1$$

Colloidal density uniform

$$\eta = \delta\left(\frac{D}{2}\right)$$

Ferroparticles confined to central layer
Colloidal *segregation*

MORE TECHNICAL STUFF

FIGURES OF MERIT

- θ_0 Nematic director distortion on centre line
- ψ_0 Ferroparticle long axis distortion
- s Order parameter associated with ferroparticle segregation

SCALED FREE ENERGY

$$F = \int_0^1 \left(\frac{1}{2}\left(\frac{\partial\theta}{\partial z}\right)^2 - \eta h \sin\psi - \eta h_b \cos\psi + \eta w \sin^2(\theta - \psi) + t\eta \ln\eta \right) dz$$

(a) $h = \dfrac{\bar{f} M_s H D^2}{K}$ scaled magnetic field

(b) $w = \dfrac{\bar{f} W_p D^2}{K} \approx \bar{f} \dfrac{D^2}{\zeta d}$ scaled ferroparticle - nematic coupling

$\zeta = \dfrac{K}{W}$ is anchoring extrapolation length

(c) $t = \dfrac{k_B T \bar{f} D^2}{\nu K}$ scaled temperature

METHODS

- **Asymptotic analysis**

  (Landau-type expansion)
- **Numerical solutions**

  (MATLAB, sometimes using inbuilt routines, sometimes not)

Euler-Lagrange equations

$$\frac{1}{2}\theta_{zz} - \eta w \sin\left(2\left(\theta - \psi\right)\right) = 0$$

$$h \cos\psi + \eta w \sin\left(2\left(\theta - \psi\right)\right) = 0$$

$$h \sin\psi = w \sin^2\left(\theta - \psi\right) + t\left(\ln\eta + 1\right) + \lambda$$

$$\vartheta\left(0\right) = \vartheta\left(1\right) = 0$$

$$\int_0^1 \eta\left(z\right) dz = 1$$

RESULTS

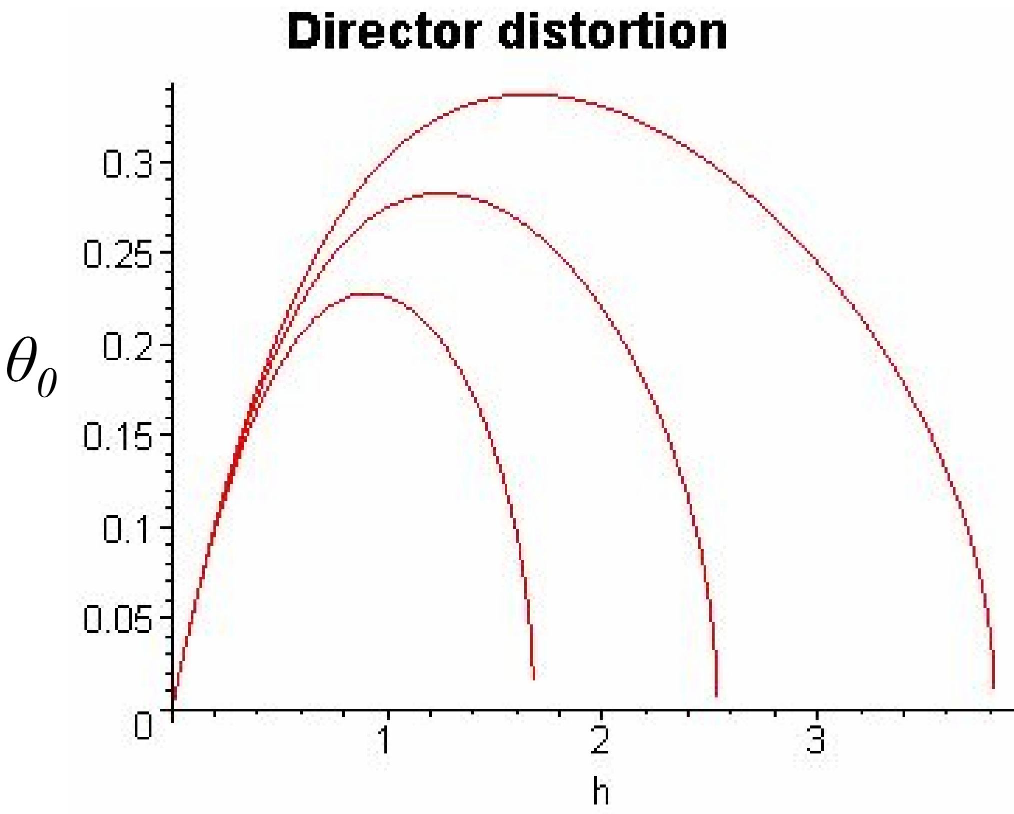
<u>High temperature regime</u> (*t* >> *1*)

No ferroparticle segregation

<u>Three regimes</u>:

- **Weak coupling** $w < w_c = \pi^2/2$
- **Intermediate coupling** $w_c < w < 4w_c/3$
- **Strong coupling** $w > 4w_c/3$

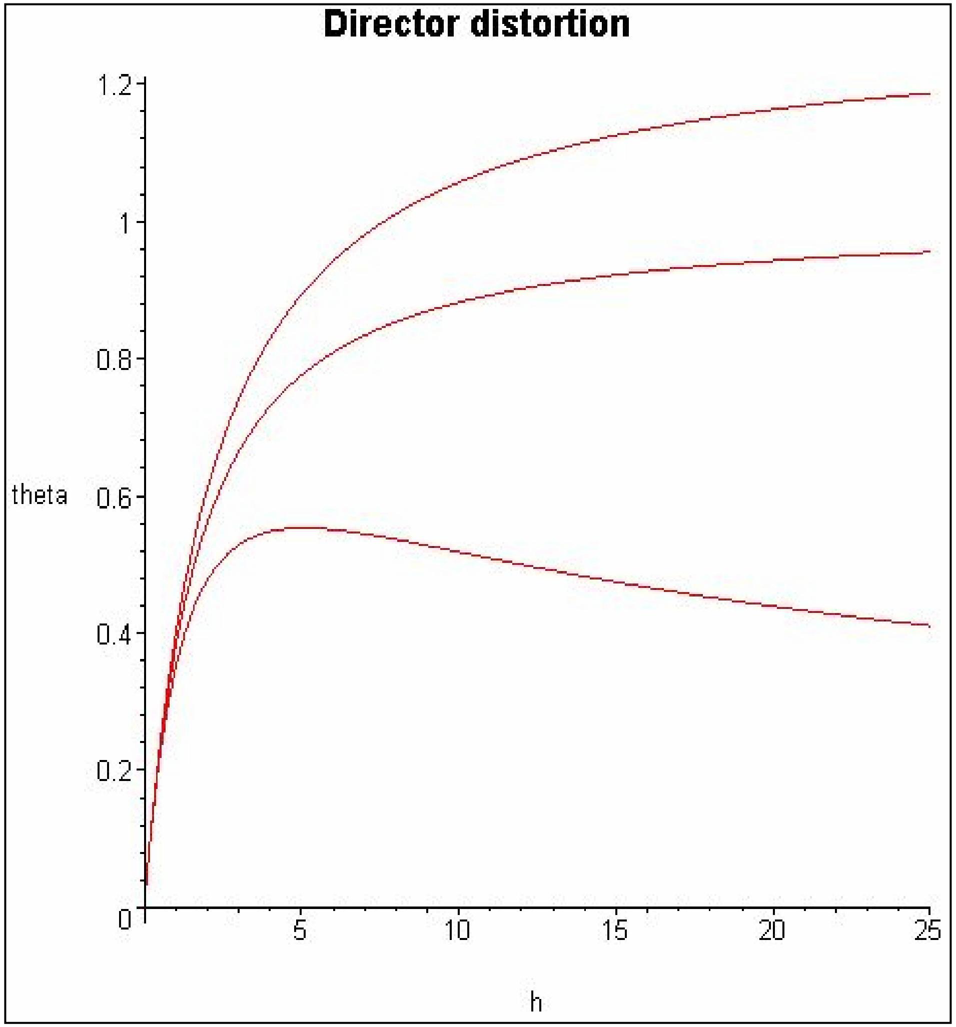
Weak coupling $w < w_c = \pi^2/2$



- ψ_0 monotonic with h
- ψ_0 saturates at h >>1
- $\theta_0(h)$ non-monotonic
- No Frederiks threshold
- Reentrant Frederiks transition at $h=h_c$

$$h_c^{-1} = 2\left(w_c^{-1} - w^{-1}\right)$$

Higher coupling



Strong coupling

$w > 4w_c/3$

$\theta_0(h)$ now monotonic

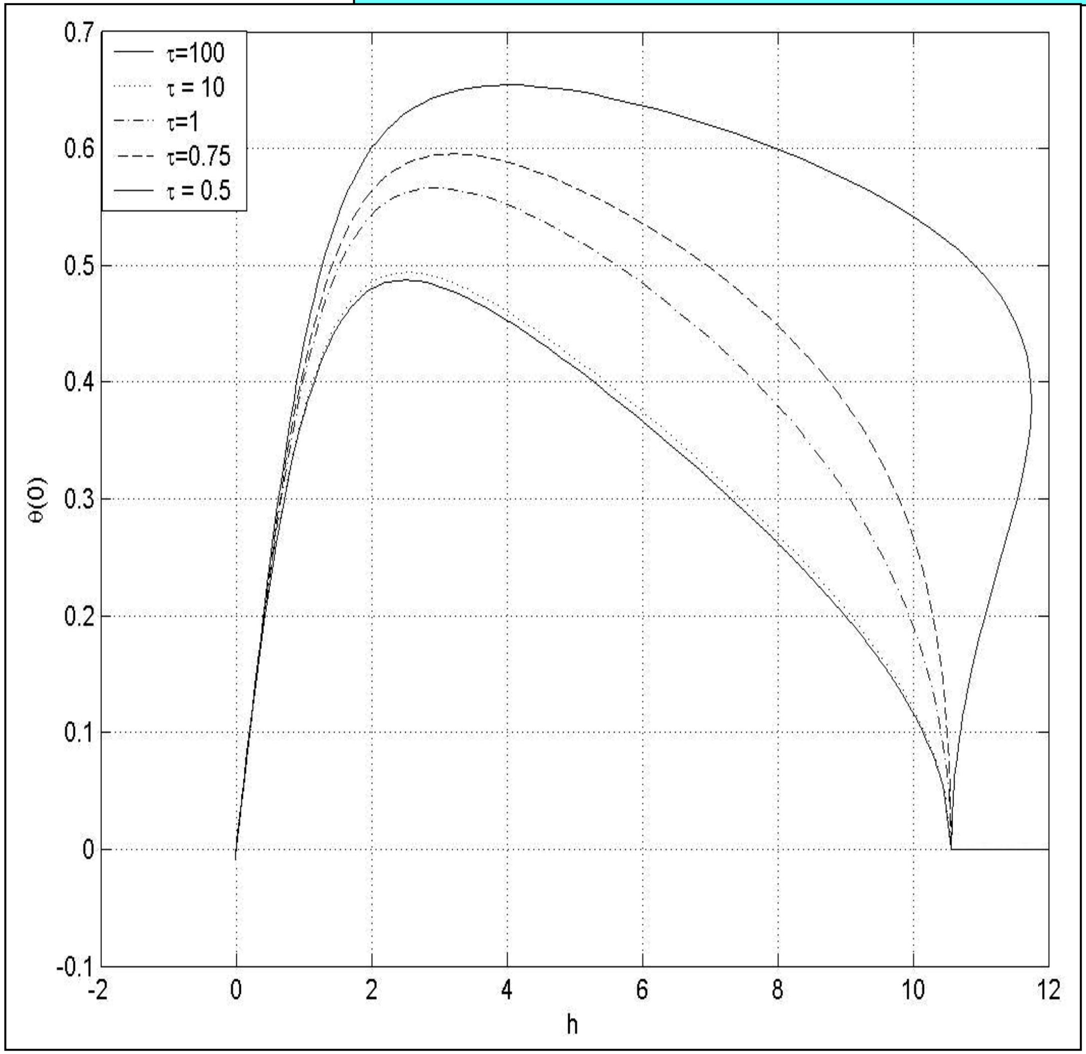
Saturates for high w

Intermediate coupling

$w_c < w < 4w_c/3$

$\theta_0(h)$ still non-monotonic

Temperature effects



$w < w_c$

Director response

Frederiks transition

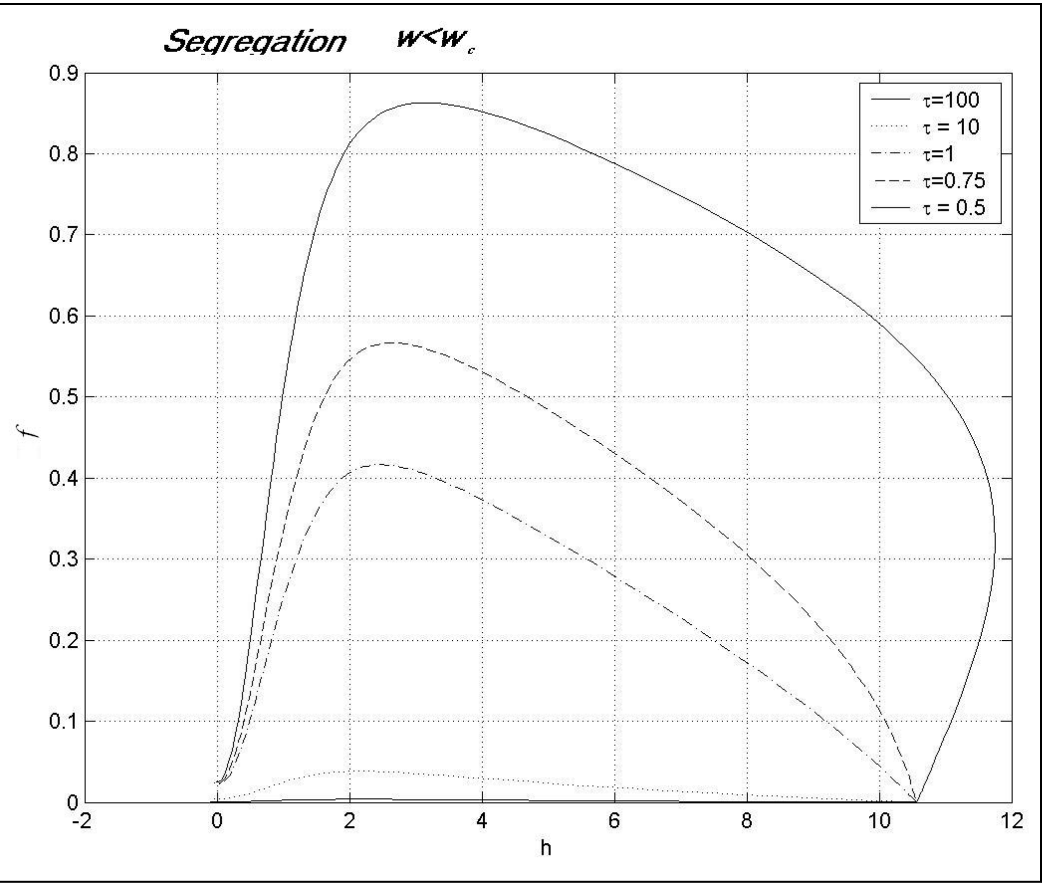
first order

Tricritical point at

$t \approx 0.7$

Temperature effect (2)

Segregation

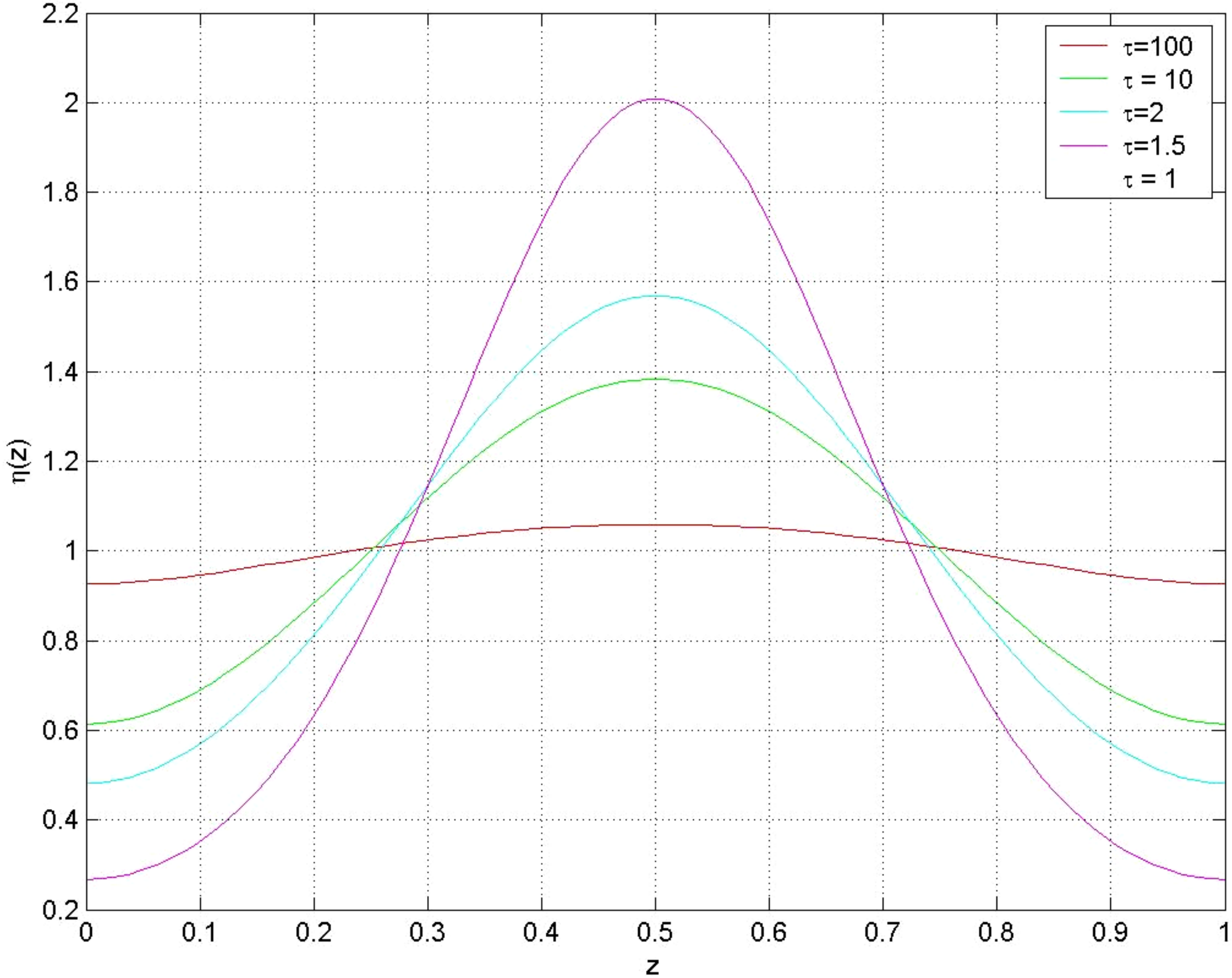


$w<w_c$

Segregation

Segregation is large when magnetic response is large in centre of cell, but small at edge.

Temperature-induced segregation



$w < w_c$

$h < h_c$

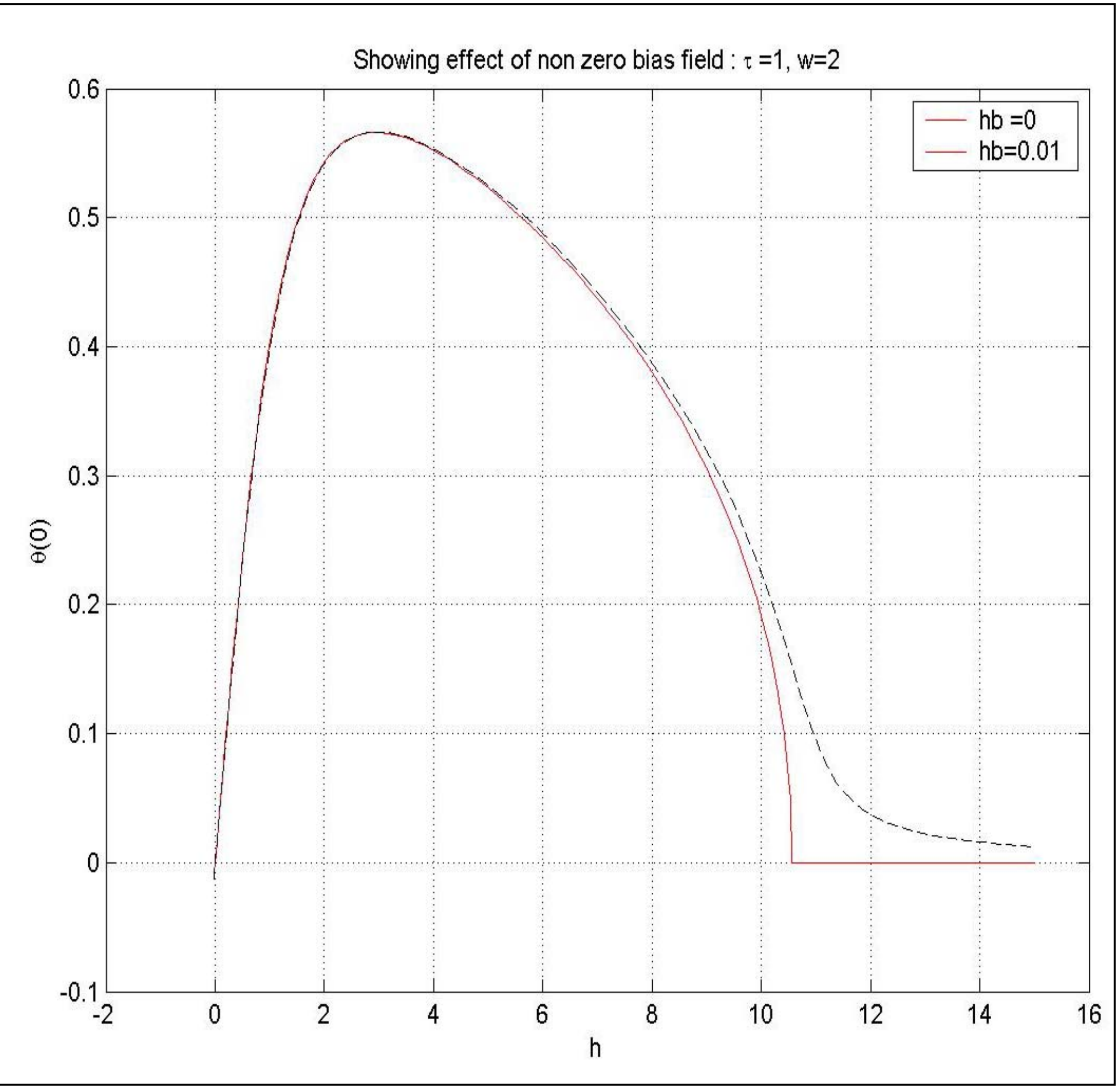
Weak coupling

Segregation order parameter

$$s = -\int_0^1 dz\,\eta(z)\cos 2\pi z$$

$\eta(z)=1$	$\Rightarrow$	$s=0$	no segregation
$\eta(z)=\delta(½)$	$\Rightarrow$	$s=1$	perfect segregation

BIAS FIELD

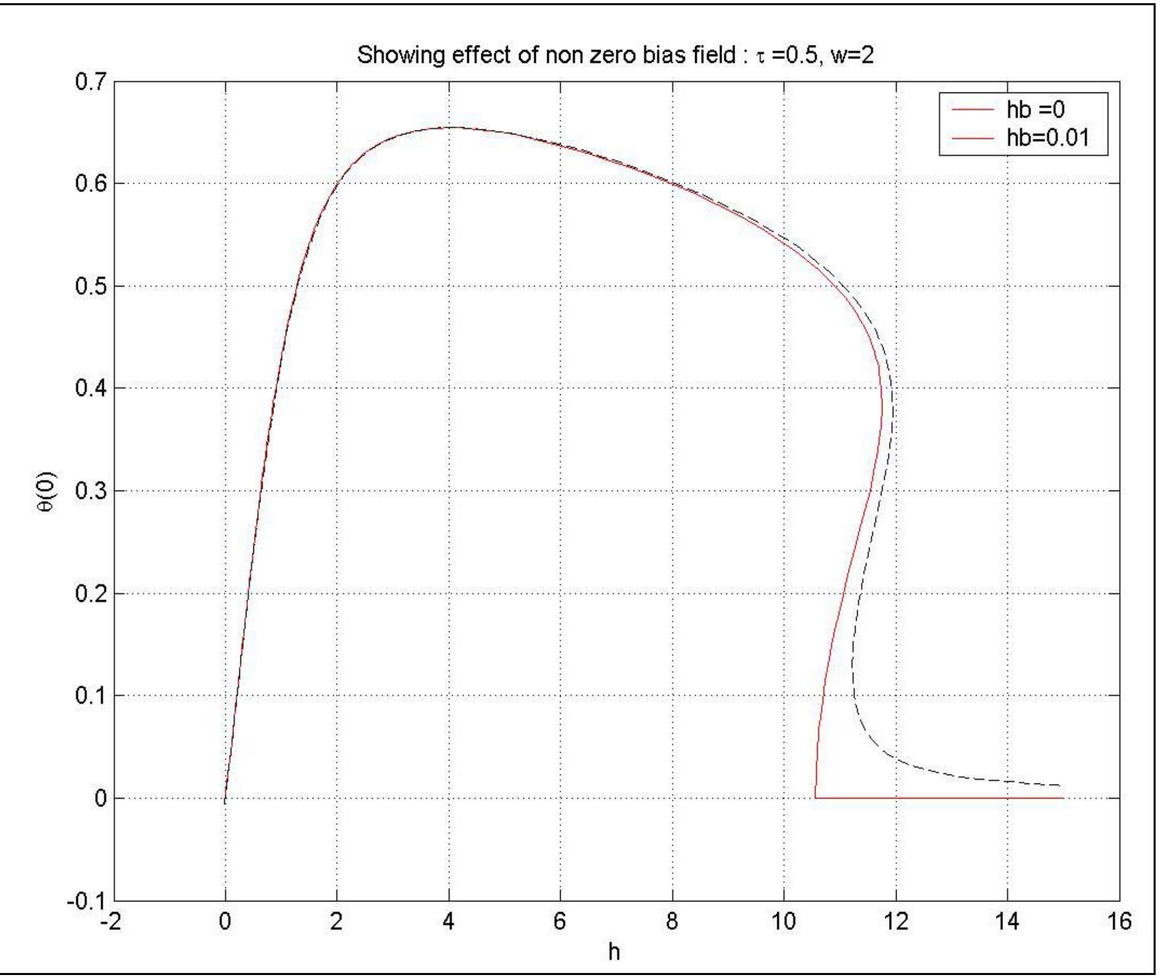


Frederiks transition rounded

Response finite at finite field

$$\theta_0 \propto \frac{h_b}{h-2w}$$

BIAS FIELD (2)



Finite temperature

and

Bias Field

Van der Waals loop

Temperature-induced segregation

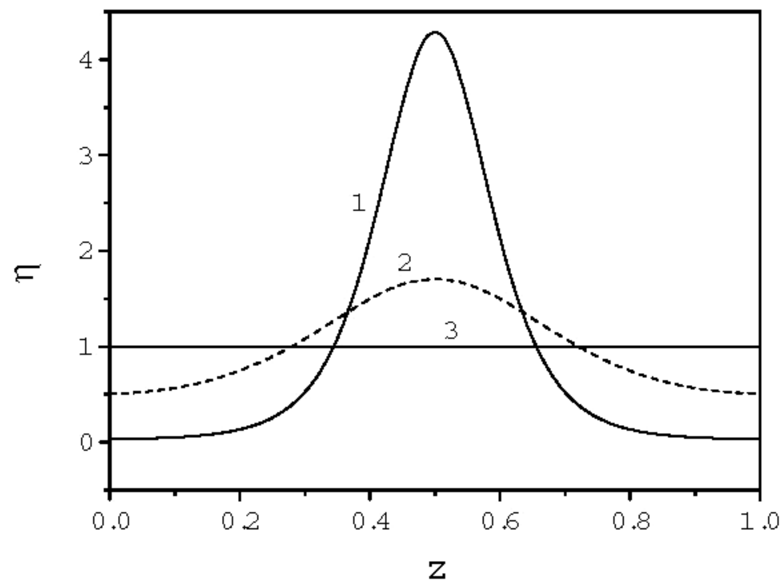
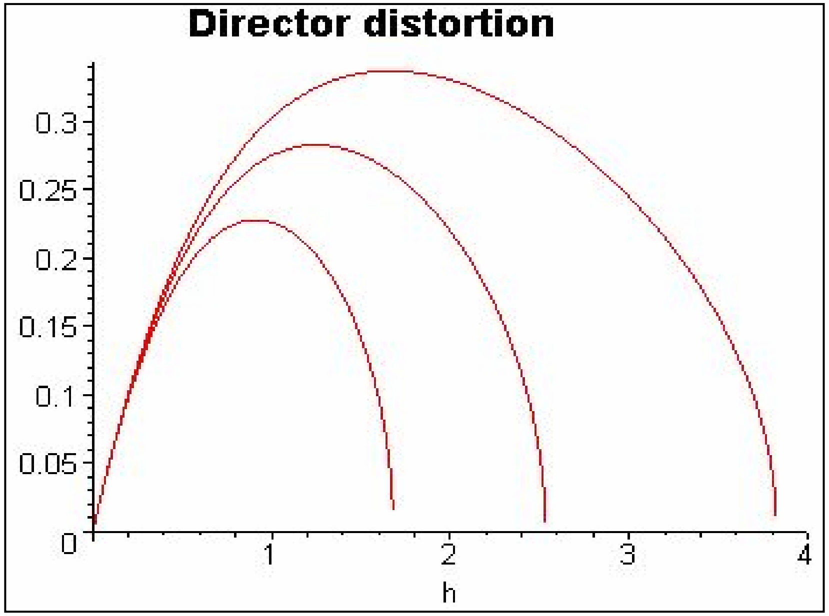


FIG. 8. Concentration profiles at $t = 0.1$, $w = 2$ and $h = 7.5$, $h_b = 0.01$. Curves 1-3 correspond to the upper, middle (unstable) and lower parts of the van der Waals loop, respectively.

Why first order?

$$F = \int_0^1 \left(\frac{1}{2}\left(\frac{\partial \theta}{\partial z}\right)^2 - \eta h \sin\psi - \eta h_b \cos\psi + \eta w \sin^2(\theta - \psi) + t\eta \ln \eta \right) dz$$



Stability analysis with respect to ψ close to upper critical field h_c.

Coupling between η, t, w

Segregation order parameter reminder

$$s = -\int_0^1 dz\,\eta(z)\cos 2\pi z$$

$\eta(z)=1$	$\Rightarrow$	$s=0$	no segregation
$\eta(z)=\delta(\frac{1}{2})$	$\Rightarrow$	$s=1$	perfect segregation

Why first order?

Coupling of segregation and orientational order parameters

$$F \sim -\frac{1}{2}(a-a_c)\psi^2 + \frac{1}{4}b\psi^4 + \ldots - \varepsilon\psi^2 s + \frac{d}{2}ts^2$$

$$\varepsilon = 0 \Rightarrow \quad \psi = \sqrt{a-a_c/b}$$

$$\varepsilon \neq 0 \Rightarrow s = \frac{\varepsilon\psi^2}{t}$$

$$F \sim -\frac{1}{2}(a-a_c)\psi^2 + \left(\frac{1}{4}b - \frac{\varepsilon^2}{2dt}\right)\psi^4$$

Fourth order term negative

$\Rightarrow$

Discontinuous transition

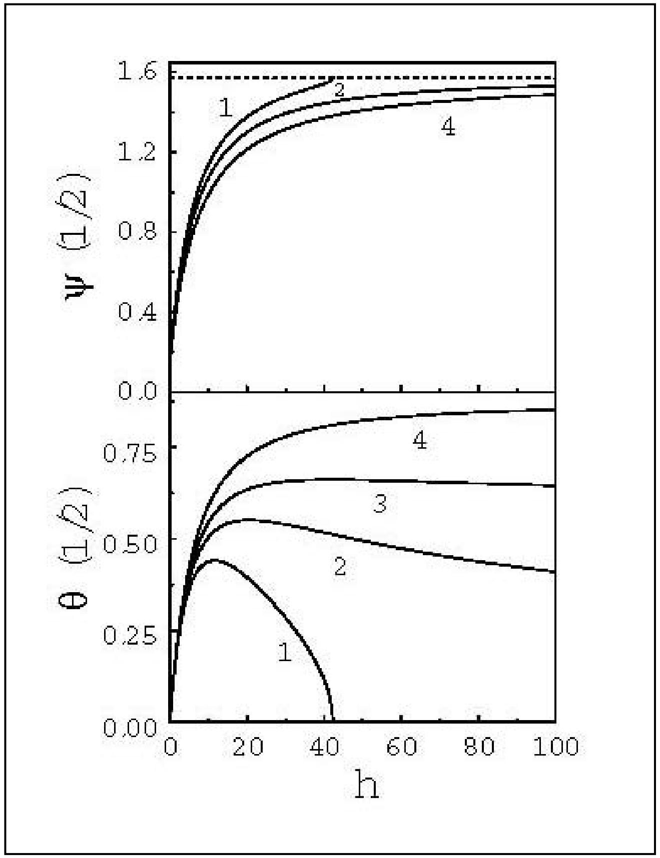
Requires low temperature

Critical temperature

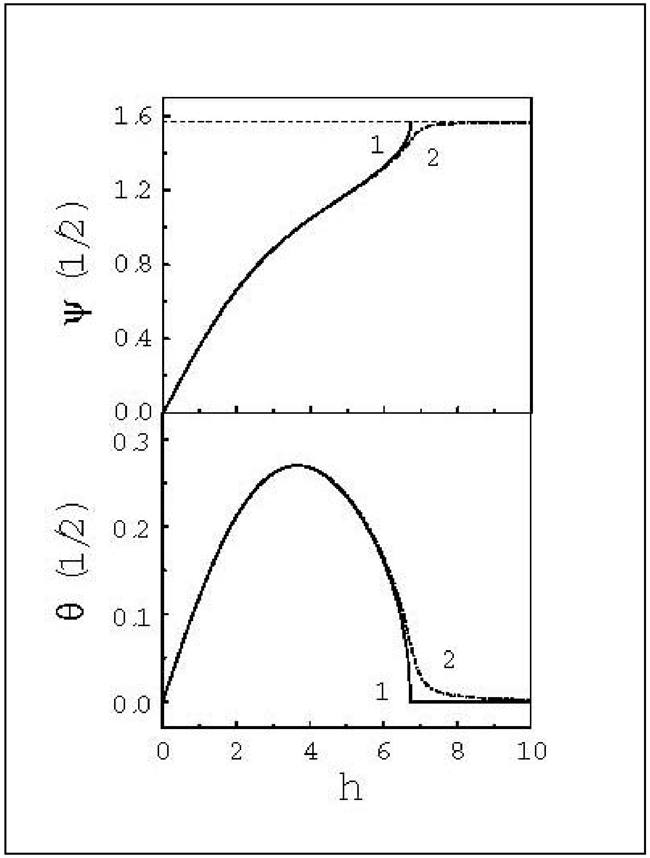
First-order Freedericksz transition at $t<t_c$

$$t_c(w) \cong \left(\frac{w_c}{2}\right)\left(\frac{w}{w_c}\right)^3$$

SUMMARY

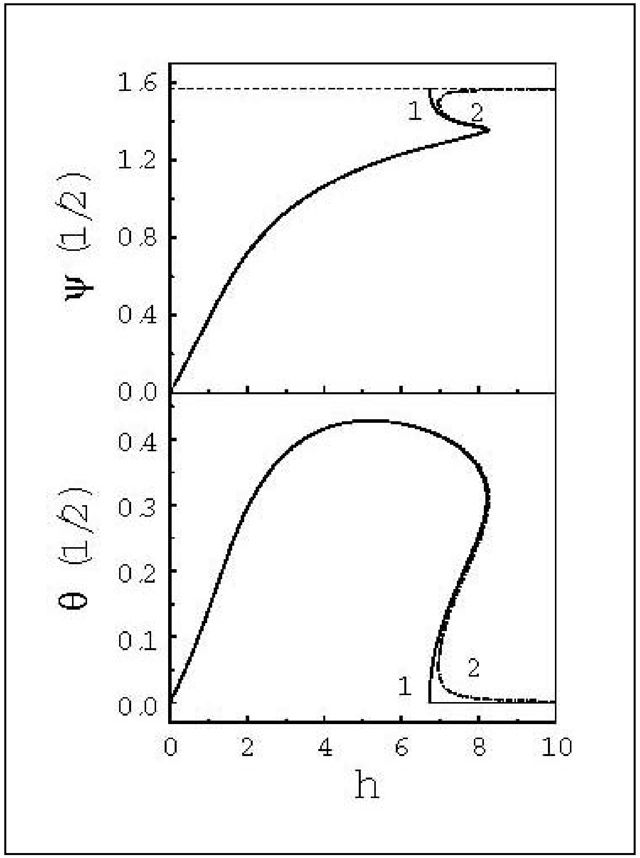


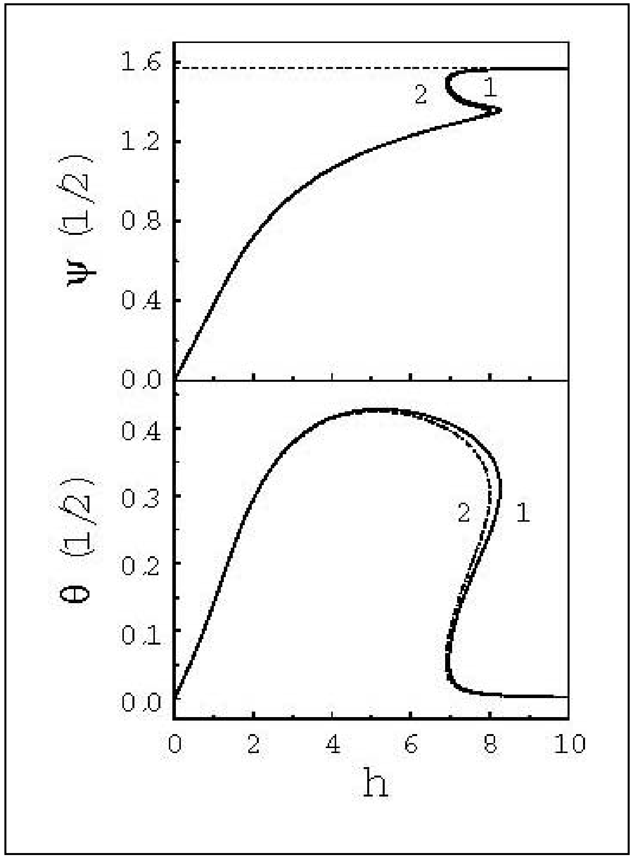
Below w_c no bias field



Below w_c bias field

SUMMARY (2)





Below w_c bias field
Finite temperature

Some Numbers (5CB)

($K \sim 5\times10^{-7}$ dynes), T ~ 25 C

D ~150μ (cell width) W ~ 3 $\times 10^{-3}$ ergs

f ~ *5* $\times 10^{-6}$ (*i.e.* very low) ζ ~ *1* μ (very weak anchoring)

L ~ *0.1* μ (particle length) *d* ~ *0.03* μ (particle length)

L, d < ζ

anchoring can be averaged, director can be taken as uniform

No problems with Saturn rings or dipoles.

Particles: Fe_3O_4 (magnetite)

Surfactant coating …. oxy ethyl propylene glycol

reduces anchoring and prevents coagulation

Some Numbers (5CB)

For this system:

w~2 (in weak coupling regime, but close to borderline)

t ~1 (in high temperature perturbation regime, picture OK)

H_c ~ 65 Oersted

h ~1 equivalent to H ~10 Oersted

Decrease *f* but increase *D* to compensate will give same scaling.

If anchoring stronger, then move into strong coupling regime, unless *f* reduced, but then *t* low and strong segregation occurs.

EXPERIMENTS IN PROGRESS

SIGNIFICANCE OF OUR WORK

- Systematic and complete theory over full range of h,w,t
- Important non-dimensional parameters identified
- Stable ferrocolloids now available
- Parameter regime reasonable
- Fruitful to seek device applications

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