Università di Pavia
Facoltà di Ingegneria
Master Course in Building Engineering and Architecture
Italian Chinese Curriculum

Written Test of Analytical Mechanics

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SURNAME **NAME**

Problem 1

Given the following curve in the three–dimensional Euclidean space

$$p(t) - o = 2t\cos t\boldsymbol{e}_x + \frac{1}{2}t^2\boldsymbol{e}_y - \mathrm{e}^{-t}\boldsymbol{e}_z,$$

find the binormal unit vector $\mathbf{b}$ at the point p corresponding to $t = 0$.

Problem 2

A beam of length ℓ is simply supported at its ends O and A as shown in Fig. 1. An external load is applied with density per unit length

$$\mathbf{f}(s) = -\alpha\sin\left(\frac{s}{\ell}\pi\right)\boldsymbol{e}_y, \qquad \alpha > 0,$$

where s represents the arclength, $0 \le s \le \ell$. The internal couple stress is given by $\boldsymbol{\Gamma} = Bc\mathbf{b}$, where B is the bending rigidity, c is the curvature and $\mathbf{b}$ is the binormal unit vector of the deformed shape.

At equilibrium,

- a. find the shape of the deformed beam, under the assumption of small deflections;
- b. find the maximum displacement $|y_{max}|$ and the maximum deflection $|\theta_{max}|$;
- c. under what condition on α and B can we assume small deflections?
- d. find $\boldsymbol{\Gamma}$;
- e. find the stress vector $\boldsymbol{\Phi}$ and the supporting forces $\boldsymbol{\Phi}_{\mathsf{O}}$, $\boldsymbol{\Phi}_{\mathsf{A}}$ at O and A, respectively;
- f. check the total balance of forces and torques.

Problem 3

In a vertical plane, a cable $\overset{\frown}{\mathsf{AB}}$ with mass density λ and length $2\sqrt{3}\ell$ has its ends A and B constrained to slip with no friction along two guides, one vertical and the other horizontal (see Fig. 2).

An external force $\mathbf{f}$ is applied at B

$$\mathbf{f} = 2\lambda g\ell\boldsymbol{e}_x.$$

At equilibrium,

- a. find the value of $|\mathsf{OB}|$;
- b. find the shape of the cable $y = y(x)$;
- c. find the total external force $\boldsymbol{\Phi}_{\mathsf{A}}$ at A;
- d. find the total external force $\boldsymbol{\Phi}_{\mathsf{B}}$ at B;
- e. find the tension $\tau = \tau(x)$;
- f. find the value of $|\mathsf{OA}|$.