Quali fra le seguenti sono liste di vettori linearmente <u>indipendenti</u>?

$$\left\{ \underline{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \underline{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \underline{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 2 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ -2 \\ 5 \\ 0 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Quali sono generatori di $\mathbb{R}^3$?

$$S_1 = \left\{ \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}2\\0\\2\end{pmatrix}, \begin{pmatrix}3\\-1\\3\end{pmatrix}, \begin{pmatrix}0\\2\\0\end{pmatrix} \right\}$$

$$S_2 = \left\{ \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$$

$$S_3 = \left\{ \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\2\\2\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix} \right\}$$

$$S_4 = \left\{ \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\}$$

$$S_5 = \left\{ \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}2\\2\\0\end{pmatrix} \right\}$$

$$S_6 = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$$

$$S_7 = \left\{ \begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}2\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$$

$$S_8 = \left\{ \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}, \begin{pmatrix}4\\0\\2\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\0\\4\end{pmatrix} \right\}$$

$$S_9 = \left\{ \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix} \right\}$$

Determinare una base per ciascun sottospazio

$\mathrm{Span}(S_i) = \mathrm{Span}(\underline{v}_1 \dots \underline{v}_k)$ $\qquad i = 1 \dots 9$

se $S_i = \{\underline{v}_1 \dots \underline{v}_k\}$

mediante algoritmo di estrazione

Dire se i seguenti insiemi sono generatori di $\mathbb{R}^4$

$$\mathcal{S}_1=\left\{\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}1\\-1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\2\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\right\}$$

$$\mathcal{S}_2=\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}1\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\\-1\end{pmatrix}\right\}$$

$$\mathcal{S}_3=\left\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix},\begin{pmatrix}0\\1\\1\\0\end{pmatrix},\begin{pmatrix}1\\2\\2\\1\end{pmatrix},\begin{pmatrix}2\\1\\1\\2\end{pmatrix}\right\}$$

$$\mathcal{S}_4=\left\{\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}\right\}$$

$$\mathcal{S}_5=\left\{\begin{pmatrix}1\\1\\0\\0\end{pmatrix},\begin{pmatrix}1\\1\\2\\2\end{pmatrix},\begin{pmatrix}1\\1\\2\\-2\end{pmatrix},\begin{pmatrix}1\\1\\3\\4\end{pmatrix},\begin{pmatrix}0\\0\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\0\\1\end{pmatrix}\right\}$$

$$\mathcal{S}_6=\left\{\begin{pmatrix}1\\1\\0\\0\end{pmatrix},\begin{pmatrix}2\\2\\1\\1\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\2\\3\end{pmatrix}\right\}$$

$$\mathcal{S}_7=\left\{\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\1\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\right\}$$

$$\mathcal{S}_8=\left\{\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\right\}$$

Determinare una base per ognuno dei sottospazi generati mediante algoritmo di estrazione

Per brevità, indichiamo con

$\text{span}(\mathcal{S})$

lo span dei vettori di $\mathcal{S}$.

$\mathcal{S} = \{ \underline{v}_1 \dots \underline{v}_k \}$ $\quad \text{span}(\mathcal{S}) = \text{span}(\underline{v}_1, \dots \underline{v}_k)$

Calcolare $\dim(\text{Span}(\mathcal{S}_i))$ per gli insiemi precedenti.

- Determinare una base per:

$\text{Span}(\mathcal{S}_3) \cap \text{Span}(\mathcal{S}_5)$

$\text{Span}(\mathcal{S}_3) + \text{Span}(\mathcal{S}_5)$

$\text{Span}(\mathcal{S}_3) \cap \text{Span}(\mathcal{S}_6)$

$\text{Span}(\mathcal{S}_3) + \text{Span}(\mathcal{S}_6)$

$\text{Span}(\mathcal{S}_5) \cap \text{Span}(\mathcal{S}_6)$

$\text{Span}(\mathcal{S}_5) + \text{Span}(\mathcal{S}_6)$

- Per quale motivo $\mathcal{S}_4$ <u>non</u> è una lista di generatori di $\mathbb{R}^4$?

- Fornire le equazioni cartesiane dei sottospazi $\text{Span}(\mathcal{S}_3)$, $\text{Span}(\mathcal{S}_5)$, $\text{Span}(\mathcal{S}_6)$

ponendo $\underline{X} = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ per il vettore generico di $\mathbb{R}^4$

- Verificare che lo spazio intersezione si ottiene mettendo a sistema le equazioni cartesiane corrispondenti.

Sia

$\mathcal{B} = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ Base di V, sp. vett. reale, f. gen.

$\{\underline{v}_1+\underline{v}_2, \underline{v}_2+\underline{v}_3, \underline{v}_3+\underline{v}_1\}$ è ancora base?

$\{\underline{v}_1, \underline{v}_1+\underline{v}_2, \underline{v}_1+\underline{v}_2+\underline{v}_3\}$?

$\{v_2-v_1, v_3-v_2, v_1-v_3\}$?

$\{\underline{v}_1, \underline{v}_2, \underline{v}_1+\underline{v}_2\}$?

$\{\underline{v}_3, \underline{v}_1, \underline{v}_2\}$?

$\{\underline{v}_1, 2\underline{v}_2, 3\underline{v}_3\}$?

$\{\underline{v}_1+2\underline{v}_2+3\underline{v}_3, \underline{v}_1+\underline{v}_2+\underline{v}_3, \underline{v}_2+2\underline{v}_3\}$?

$\{\underline{v}_1+3\underline{v}_2, \underline{v}_2+3\underline{v}_3, \underline{v}_3+3\underline{v}_1\}$?

$\{\underline{v}_1+\underline{v}_2, \underline{v}_2+\underline{v}_3, \underline{v}_1+2\underline{v}_2+\underline{v}_3\}$?

$\{v_1+v_3, v_3+v_2, \underline{v}_1-\underline{v}_2\}$

$\{\underline{v}_1, \underline{v}_2\}$?

Sia V uno spazio vettoriale reale, finit. generato; $n = \dim V$

$\{\underline{v}_1, \underline{v}_2 \cdots \underline{v}_n\} = \mathcal{B}$ una base di V

Quali fra i seguenti insiemi sono ancora base di V?

$$\{\underline{v}_1,\ \underline{v}_1+\underline{v}_2,\ \underline{v}_1+\underline{v}_2+\underline{v}_3,\ \underline{v}_1+\underline{v}_2+\underline{v}_3+\underline{v}_4,\ \cdots,\ \underline{v}_1+\underline{v}_2+\cdots+\underline{v}_n\}$$

$$\{v_1,\ \underline{v}_2+\underline{v}_1,\ \underline{v}_3+\underline{v}_2,\ \underline{v}_4+\underline{v}_3,\ \cdots\ \underline{v}_n+\underline{v}_{n-1}\}$$

$$\{\underline{v}_1+\underline{v}_n,\ \underline{v}_1-\underline{v}_n,\ \underline{v}_2+\underline{v}_{n-1},\ \underline{v}_2-\underline{v}_{n-1}\ \cdots,\ \underline{v}_{\frac{n}{2}+1}-\underline{v}_{\frac{n}{2}}\}$$

(n pari)

$$\{\underline{v}_1-\underline{v}_n,\ \underline{v}_2-\underline{v}_1,\ \underline{v}_3-\underline{v}_2,\ \underline{v}_4-\underline{v}_3,\ \cdots\ \underline{v}_n-\underline{v}_{n-1}\}$$

$$\{\cancel{\underline{v}_n-\underline{v}_1,\ \underline{v}_2}$$

$$\{\cancel{\underline{v}_n\ \underline{v}_1 - n\underline{v}_n},$$

$$\{\ -\underline{v}_1,\ v_1-2v_2,\ v_2-3\underline{v}_3,\ \cdots\ v_{n-1}-n\underline{v}_n\}$$

$$\{\underline{v}_1-\underline{v}_2,\ \underline{v}_2-\underline{v}_3,\ \underline{v}_3-\underline{v}_4,\ \cdots\ \underline{v}_{n-1}-\underline{v}_n\}$$

Es.

a) Sia $\underline{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \in \mathbb{R}^3$

considerare la Base di $\mathbb{R}^3$, $B = \{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$

Con $\underline{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Determinare le coordinate di $\underline{v}$ sulla base B

$$\left([\underline{v}]_B = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right)$$

b) Ripetere la richiesta per i vettori ~~sulle~~

$\underline{w}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad \underline{w}_2 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ $\quad \underline{w}_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$[\underline{w}_1]_B = ?$ $\quad [\underline{w}_2]_B = ?$ $\quad [\underline{w}_3]_B = ?$

c) Ripetere le richieste per il vettore $\underline{v}$ sulle basi

$$B_1 = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\} \qquad B_2 = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$B_3 = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$B_4 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right\}$$